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Letter to the Editor

Comments on "Aerodynamic noise generation by a stationary body in a turbulent air stream"

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1. Introduction

Bies et al. [1] used an experimental arrangement to investigate the noise radiated by an acoustically compact block suspended in a non-uniform fluctuating flow, i.e., a vortex wake. The measured radiated sound power was compared to that predicted from the resulting force on the block, using the theory [2] of an oscillating sphere, and Curle's equation [3] reduced to the case of a concentrated hydrodynamic force. The present letter intends to clarify the difference between these two theories, and outline the theory that applies to the described experiment.

2. The oscillating sphere theory

To clarify how the case presented in Ref. [1] should be theoretically approached, it is useful to start with the case of an oscillating sphere, as first analysed by Morse [2], and then by others, including Pierce [4]. Consider a sphere of radius a, oscillating along the z-axis, with a velocity $\overrightarrow{U_z} \bullet \overrightarrow{e_z} = \mathrm{Re}(U_z \mathrm{e}^{-\mathrm{i}\omega t})$ in a fluid where the sound velocity is c_0, and the density is ρ_0. Introducing the wavenumber $k_0 = \omega/c_0$ (ω is the radian frequency), the pressure p radiated by the oscillating sphere is

$$p = \frac{\mathrm{i}\omega\rho_0 a^3 U_z}{2} \cos\phi \frac{(\mathrm{i}k_0 - 1/r)}{1 - (k_0 a)^2/2 - \mathrm{i}k_0 a} \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0(r-a))}}{r}, \tag{1}$$

where negative time dependance $\mathrm{e}^{-\mathrm{i}\omega t}$ is used, r is the distance from the source to the observer, and ϕ is the angle measured from the axis of oscillation. This can be approximated at low wavenumbers ($k_0 a \ll 1$) and in the far field

$$p \cong -\frac{c_0 k_0^2 \rho_0 a^3}{2} U_z \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0(r-a))}}{r}. \tag{2}$$

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Finally, Pierce also calculates the radiated acoustic power W from the farfield approximation of the pressure

$$W = 4\pi a^2 \rho_0 c_0 \frac{1}{3}\overline{|U_z(t)|^2}\left[\frac{(k_0 a)^4}{4 + (k_0 a)^4}\right] \tag{3}$$

with the overbar denoting time average. At low frequency ($k_0 a \ll 1$), this can be approximated as

$$W \cong \frac{\pi a^2 \rho_0 c_0}{3}\overline{|U_z(t)|^2}. \tag{4}$$

Here, the acoustic field is expressed as a function of U_z. Since the whole pressure field is caused by the motion of the sphere, it can also be expressed as a function of the resulting force F on the sphere, since F and U_z are interdependent.

Integration of the pressure over the surface S of the sphere yields the resulting force

$$F = \iint_S p \, \mathrm{d}S = -\frac{2\pi\omega\rho_0 a^3}{3} \frac{\mathrm{i} + k_0 a}{1 + (k_0 a)^2/2 - \mathrm{i}k_0 a} U_z. \tag{5}$$

Expressing U_z as a function of F and substituting in the expression for the radiated acoustic power, at low frequency ($k_0 a \ll 1$)

$$\begin{aligned} W &\cong \frac{3\pi f^2}{2\rho_0 c_0^3}|F|^2, \quad k_0 a \ll 1 \\ &= \frac{3\pi f^2}{\rho_0 c_0^3}\overline{|F(t)|^2} \quad \text{if } F(t) = |F|\mathrm{e}^{-\mathrm{i}\omega t}, \end{aligned} \tag{6}$$

where f is the frequency. Note that this low wavenumber approximation can also be derived more simply by writing the pressure radiated by a dipole, and calculating the resulting force on the sphere, as indicated by Bies et al. [1].

In the case of an oscillating sphere in a medium at rest, the surface pressure and velocity on the sphere are related to each other by the boundary condition that embodies the conservation of momentum at the surface: $\mathrm{i}\omega\rho_0 u_r = \partial p/\partial r$, u_r is the radial velocity. The surface pressure is the reaction of the fluid to the motion of the sphere. In other words, the surface pressure and velocity are related to the same physical phenomenon, the motion of the sphere. It is then possible to relate the radiated acoustic field to either the fluctuating surface pressure, or the fluid motion induced by the motion of sphere, depending on how the equation is written.

In the case of a sphere that is moving in a non-uniform fluid flow, the surface pressure and velocity are not totally interdependent anymore, and the above theory does not apply. The equation developed by Curle [3] takes account of the motion of both the sphere and the fluid.

3. Curle's equation

Blake [5] rewrites Curle's equation [3] as the sum of three terms: "the radiation from the turbulent domain (1), radiation due to the instantaneous contiguous motion with phase cancellation included (2), and radiation from a distribution of forces acting on the region (3)".

The density fluctuation at time t and position $\vec{x}$ is written as

$$4\pi c_0^2(\rho(\vec{x},t)-\rho_0) = \underbrace{\frac{\partial^2}{\partial x_i \partial x_j}\int\int\int_V \frac{[T_{ij}]}{r}\,\mathrm{d}V(\vec{y})}_{(1)} - \underbrace{\int\int_S \frac{l_i}{r}\left[\frac{\partial(\rho u_i)}{\partial t}\right]\mathrm{d}S(\vec{y})}_{(2)}$$
$$+\underbrace{\frac{\partial}{\partial x_i}\int\int_S \frac{l_j}{r}[\rho u_i u_j + \tau'_{ij} + p\delta_{ij}]\,\mathrm{d}S(\vec{y})}_{(3)}, \tag{7}$$

where T_{ij} is Lighthill's stress tensor

$$T_{ij} = \rho u_i u_j + (p - c_0^2\rho)\delta_{ij} + \tau'_{ij}, \tag{8}$$

where u is the velocity disturbance, τ'_{ij} is the viscous stress tensor, V is the volume that encompasses all regions of turbulent flow, l_i are the components of the outward normal to the bounding surface S, and the brackets [] imply evaluation at the time retarded by acoustic propagation $t - r/c_0$. This equation is now applied to two examples, the concentrated hydrodynamic force and the oscillating sphere.

4. Applications

4.1. Concentrated hydrodynamic force

The most straightforward simplification of Curle's equation is to assume a low Mach number flow where the quadrupole volume source term can be neglected, and to reduce the surface source terms to a pressure contribution. At low frequencies, this simplification corresponds to an infinitely small fixed object applying a finite force on the fluid, without moving. This case, where the source term models a concentrated hydrodynamic force only, allows any acoustic effect around the object to be neglected. When applied to a concentrated hydrodynamic force [5], the right-hand side of this equation reduces to the pressure component of the third term, related to the distribution of forces

$$4\pi c_0^2(\rho(\vec{x},t)-\rho_0) = -\frac{\partial}{\partial x_i}\left[\frac{F_i}{r}\right] = \frac{1}{c_0}\frac{x_i}{r^2}\left[\frac{\partial F_i}{\partial t}\right]. \tag{9}$$

In the case of a harmonic force $F_i(t) = \mathrm{Re}(F_i \mathrm{e}^{-\mathrm{i}\omega t})$,

$$4\pi c_0^2(\rho(\vec{x},t)-\rho_0) = -\mathrm{i}F_i\frac{1}{c_0}\frac{x_i}{r}\frac{\mathrm{e}^{-\mathrm{i}(\omega t - r/c_0)}}{r}, \tag{10}$$

which rearranges to the following expression for the radiated acoustic pressure:

$$p(\vec{x},t) = -\mathrm{i}k_0 F_i\frac{x_i}{r}\frac{\mathrm{e}^{-\mathrm{i}(\omega t - r/c_0)}}{4\pi r}. \tag{11}$$

The acoustic power radiated by a concentrated hydrodynamic force is finally

$$W = \frac{\pi f^2 \langle |F_i(t)|^2 \rangle}{3\rho_0 c_0^3}. \tag{12}$$

The ideal case of the concentrated hydrodynamic force where noise is generated by an imposed fluctuating point force can be seen as the simplified case of a fixed rigid and very small object in a turbulent low Mach number flow. In this case, the noise emitted from the turbulence itself is negligible compared to that generated by the reaction pressures on the surface of the object. The surface velocity is zero, while the surface pressure is not. This is different from the oscillating sphere, where the surface pressure fluctuation is caused by the surface motion. As a result, by comparing Eqs. (6) and (12), for the same resulting force applied on the sphere, the radiated acoustic power is nine times smaller than that generated by an oscillating sphere as previously studied. This discrepancy is now analysed following Blake's approach [5] of the oscillating sphere, using Curle's equation.

4.2. Curle's equation applied to the oscillating sphere

Blake [5] considers the field radiated by an oscillating sphere moving along the z-axis with the speed U_z defined by

$$U_z(t) = \mathrm{Re}(U_z \mathrm{e}^{-\mathrm{i}\omega t}).$$

Curle's simplified equation then contains another source term related to the direct motion induced by the sphere as well as the pressure term induced by the fluid's reaction to the oscillating motion. The two resulting acoustic pressure fields are noted p_u and p_p, respectively

$$\begin{aligned} 4\pi p(\vec{x}, t) &= -\iint_\Sigma \frac{l_i}{r}\left[\frac{\partial(\rho u_i)}{\partial t}\right] \mathrm{d}S(\vec{y}) + \frac{\partial}{\partial x_i}\iint_\Sigma \frac{l_i}{r}[p]\, \mathrm{d}S(\vec{y}) \\ &= 4\pi p_u(\vec{x}, t) + 4\pi p_p(\vec{x}, t) \end{aligned} \tag{13}$$

in the case of an inviscid fluid, and when the quadrupole term associated with the Reynolds stresses can also be neglected, due to the slow motion of the sphere ($U_z/c_0 \ll 1$). Blake rewrites the first, velocity term p_u as

$$\begin{aligned} 4\pi p_u(\vec{x}, t) &= -\iint_\Sigma \frac{l_i}{r}\left[\rho_0 \frac{\partial u_i(\vec{y}, t)}{\partial t}\right] \mathrm{d}S(\vec{y}) = \iint_\Sigma \frac{1}{r}\left[\rho_0 \frac{\partial u_n(\vec{y}, t)}{\partial t}\right] \mathrm{d}S(\vec{y}) \\ &= \iint_\Sigma \frac{\mathrm{i}\omega\rho_0 U_z \cos\phi(\vec{y})}{r} \mathrm{e}^{-\mathrm{i}\omega(t - r/c_0)}\, \mathrm{d}S(\vec{y}). \end{aligned} \tag{14}$$

It is crucial to note that the distance r is to be measured between the surface of the sphere and the observation point. Denoting r_0 the distance between the observation point and the centre of the sphere in its initial position, Blake approximates r as

$$r \cong r_0 - a\cos(\phi - \phi(\vec{y})) \tag{15}$$

assuming the radius a is small compared to the wavelength, i.e., $k_0 a \ll 1$. The exponential can then be expanded

$$\mathrm{e}^{-\mathrm{i}\omega(t - rc_0)} \cong \mathrm{e}^{-\mathrm{i}\omega(t - r_0/c_0)}[1 - \mathrm{i}k_0 a\cos(\phi - \phi(\vec{y}))]. \tag{16}$$

Of the two terms now to be summed in the integral, the 0th order one does not contribute to the integral's value, only the first order in $k_0 a$ yields

$$p_u(\vec{x}, t) = \frac{-1}{3}\rho_0 c_0 k_0^2 U_z a^3 \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0 r)}}{r}. \tag{17}$$

To evaluate the second, pressure term p_p, Blake uses the equation previously derived for a concentrated hydrodynamic force

$$p_p(\vec{x}, t) = -\mathrm{i}k_0 F_p \frac{x_i}{r}\frac{\mathrm{e}^{-\mathrm{i}(\omega t - r/c_0)}}{4\pi r}, \tag{18}$$

where F_p is the force applied on the fluid by the sphere. The sphere being acoustically compact, the fluid impedance is principally mass like. Using the added mass of the fluid $\frac{2}{3}\rho_0\pi a^3$, the force acting on the sphere is thus

$$-F_p = -\mathrm{i}\omega(\tfrac{2}{3}\rho_0\pi a^3)U_z, \tag{19}$$

which enables p_p to be written

$$p_p(\vec{x}, t) = \frac{-1}{6}\rho_0 c_0 k_0^2 U_z a^3 \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0 r)}}{r}. \tag{20}$$

Finally, the total radiated pressure is

$$\begin{aligned} p(\vec{x}, t) &= p_u(\vec{x}, t) + p_p(\vec{x}, t) \\ &= \frac{-1}{2}\rho_0 c_0 k_0^2 U_z a^3 \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0 r)}}{r}. \end{aligned} \tag{21}$$

The acoustic field radiated by an oscillating sphere is thus generated by both the reaction of the fluid on its surface, and the direct motion induced by the sphere. This expression agrees with that previously obtained for an oscillating sphere at low wavenumber (Eq. (2)).

This acoustic pressure is that radiated by an equivalent concentrated hydrodynamic force of amplitude $F_{eq} = -\mathrm{i}\omega(2\rho_0\pi a^3)U_z = 3F_p$, i.e., three times the reaction force from the fluid, so that the radiated acoustic power can be rewritten from Eq. (12):

$$W = \frac{\pi f^2}{6\rho_0 c_0^3}|\omega(2\rho_0\pi a^3)U_z|^2. \tag{22}$$

4.3. Application to a compact object suspended in a turbulent flow

To investigate the validity of Curle's equation for a concentrated hydrodynamic force against the theory of the oscillating sphere, Bies et al. [1] used an experimental arrangement based on previous work by Martin and Bies [6] on circular saw noise. This work showed that the noise was generated by the interaction of vortices shed by an upstream tooth with a downstream tooth. Bies et al. [1] used a similar arrangement with a small dense flat rectangular block suspended on taut wires in the vortex wake of a rigidly fixed upstream blade of same thickness. The principle of this arrangement is sketched in section in Fig. 1.

Considering a low Mach number for the flow and the block surface motion, the quadrupole term representing the noise generated by the vortex shedding and the turbulent flow could be

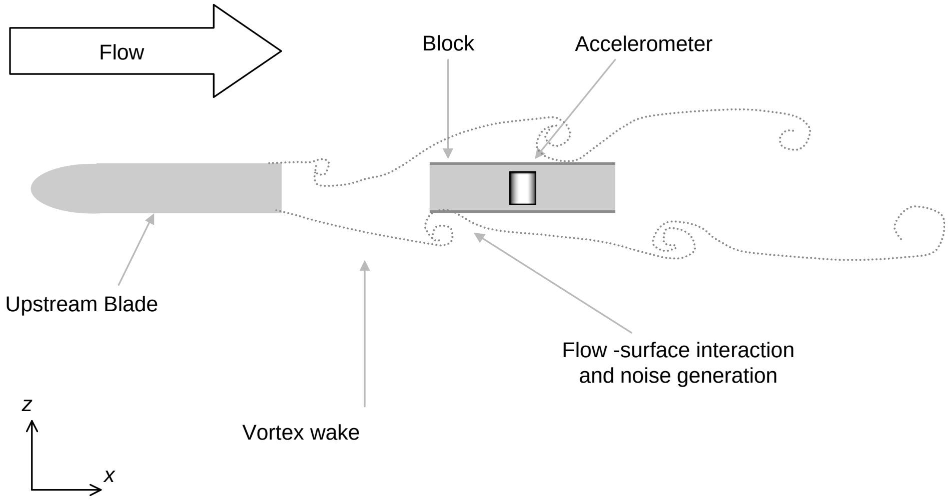

Fig. 1. Sketch of the experimental arrangement used in Ref. [1].

neglected when compared to the flow–surface interaction noise generation on the block. Curle's equation then reduced to Eq. (13) in the approximation of an inviscid fluid. It was assumed in Ref. [1] that the dominant noise contribution was from the interaction between the vortex wake and the two faces of the block that were parallel to the plane of the vortex generating blade. At low frequencies when the block was acoustically compact, the resulting force replaced the pressure surface integral. To evaluate the pressure term of Eq. (13), an accelerometer was inserted in the block, its axis perpendicular to these two surfaces. The corresponding component of the resulting force applied on the block, i.e., the surface integral of the fluctuating pressure, was deduced from the measured acceleration, using a force-to-acceleration transfer function recorded during a preliminary calibration. It should be noted that in this case, the resultant flow-induced force applied to the block was assumed to lie in the direction of the accelerometer, and that the potential contribution from the four other surfaces was neglected.

The considered problem is therefore that of a dipole noise source lying in a mean flow, its axis perpendicular to the mean flow velocity. The convective amplification effects related to the mean flow can be neglected [7] due to this relative orientation and the low Mach number flow.

In the low frequency approximation where the noise source is acoustically compact, it is possible to revert to the sphere model to simplify the analysis. Two different noise mechanisms are present on the block:

- On the one hand, its surface offers a fluctuating reaction pressure to the fluctuating flow. This surface pressure fluctuation acts as a noise source. The block being acoustically compact, the surface pressure can be integrated into a resulting force term, and the corresponding noise field is then directly related to this fluctuating concentrated hydrodynamic force, which in the present case is the opposite of the fluctuating lift force. The steady component of the surface pressure does not contribute to the radiated noise.

- On the other hand, the suspended block responds with an oscillating motion that is used to estimate the fluctuating lift force. This oscillating motion then generates noise as in the case of the oscillating sphere.

Provided that the block surface velocity is much smaller than the flow velocity, it can be assumed to have a negligible effect on the fluctuating flow. Without this feedback effect, the system can be considered as the simple superposition of two types of sources: a fluctuating concentrated hydrodynamic force and an oscillating sphere. The following analysis aims at weighing the relative importance of these two noise-generating mechanisms in the experimental case presently referred to.

For the case of a rigid acoustically compact sphere that is suspended in a turbulent flow, which corresponds to the experiment described in Ref. [1], the pressure field can be written as the sum of the pressure radiated by the oscillating sphere and the pressure radiated by the hydrodynamic force applied to the sphere

$$p(\vec{x}, t) = \frac{-1}{2} \rho_0 c_0 k_0^2 U_z a^3 \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0 r)}}{r} + \mathrm{i}k_0 F \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0 r)}}{r}, \tag{23}$$

where U_z is the vertical velocity of the sphere, and F is the force the fluid applies on it.

The motion of the obstacle is related to the force it is subjected to. If the sphere of mass m_s is only subjected to the force F imposed by the fluid flow, then $-\mathrm{i}\omega m_s U_z = F$.

Hence,

$$\begin{aligned} p(\vec{x}, t) &= \left(-\tfrac{1}{2}\rho_0 a^3 - \frac{m_s}{4\pi}\right) c_0 k_0^2 U_z \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0 r)}}{r} \\ &= -\frac{a^3}{3}\left(\tfrac{3}{2}\rho_0 + \rho_s\right) c_0 k_0^2 U_z \cos\phi \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_0 r)}}{r}, \end{aligned} \tag{24}$$

where ρ_s is the density of the sphere material. Eq. (24) shows that the relative importance of the two identified noise-generating mechanisms is fixed by the ratio between the density of the fluid and that of the object. Hence, for a steel sphere suspended in a turbulent airflow, the pressure contributed by the oscillating sphere is approximately 4000 times smaller than the force-generated pressure.

In this case, the oscillating sphere theory does not apply, as the concentrated hydrodynamic force generates the acoustic field, while the fluid motion associated with the sphere displacement has a negligible effect. On the other hand, for the case of the oscillating sphere, this surface motion term is of prime importance.

5. Conclusion

This letter considers the sound field radiated by a dense, acoustically compact rigid sphere suspended in a non-uniform flow, which corresponds to the experiment presented in Ref. [1]. It has been demonstrated that the motion of such a sphere in a turbulent air flow can be neglected when evaluating the radiated acoustic pressure, which can then be accurately estimated from the surface pressure only. The problem becomes that of a concentrated hydrodynamic force, where the motion of the object, as well as its dimensions, are not modelled.

The fluctuating force applied by the flow on a suspended compact object can be accurately estimated from its acceleration response, while its velocity does not affect the radiated acoustic field significantly. This method of estimating the applied force was used in an experimental set-up designed and used by Bies et al. [1], for which only the theory of a concentrated hydrodynamic force, as developed by Curle, applies. Experimental results of this type should therefore be compared to the results predicted from Curle's theory and not those predicted for an oscillating sphere in a stationary fluid as implied in Ref. [1].

Acknowledgements

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